Linking solar and long baseline terrestrial neutrino experiments

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We show that in the framework of three light neutrino species with hierarchical masses and assuming no fine tuning between the entries of the neutrino mass matrix, one can use the solar neutrino data to obtain information on the element U_{e3} of the lepton mixing matrix. Conversely, a measurement of U_{e3} in atmospheric or long baseline accelerator or reactor neutrino experiments would help discriminate between possible oscillation solutions of the solar neutrino problem.

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1. Currently, there are indications for neutrino oscillations in solar [1], atmospheric [2] and accelerator [3] experiments, with the strongest evidence coming from the Super-Kamiokande atmospheric neutrino data [2]. If all correct, these results would imply the existence of at least four light neutrino species, ν_e, ν_μ, ν_τ and ν_s, where ν_s is a sterile (electroweak singlet) neutrino. Of the above mentioned experimental evidence, the result of the accelerator LSND experiment is the only one that has not yet been independently confirmed. If it is excluded, the remaining solar and atmospheric neutrino anomalies can be explained through oscillations among just three standard neutrinos – ν_e, ν_μ and ν_τ. The oscillation probabilities for relativistic neutrinos then depend on two mass squared differences $\Delta m^2_{21} \equiv \Delta m^2_\odot$ and $\Delta m^2_{32} \equiv \Delta m^2_{atm}$, three mixing angles θ_{12}, θ_{13} and θ_{23}, and one CP-violating phase δ. With the parametrization of the 3×3 leptonic mixing matrix U which coincides with the standard parametrization of the quark mixing matrix [4], one can identify the mixing angle which is responsible for the dominant channel of the atmospheric neutrino oscillations with θ_{23}, the one that is primarily responsible for the solar neutrino oscillations with θ_{12} and the mixing angle which enters (along with θ_{23}) into the probabilities of the subdominant $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations of atmospheric neutrinos and long baseline $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations with θ_{13}. For the values of the neutrino parameters allowed by the data, the CP-violating effects in neutrino oscillations should be rather small, and we shall therefore omit the phase δ in our analysis.

The Super-Kamiokande atmospheric neutrino data imply $\Delta m^2_{32} \simeq (2-6) \times 10^{-3}$ eV^2, $\theta_{23} \simeq (45 \pm 12)^\circ$, and the combined data of the solar neutrino experiments lead to four domains of allowed values of Δm^2_{21} and θ_{12} corresponding to the four neutrino oscillation solutions to the solar neutrino problem – large mixing angle MSW (LMA), small mixing angle MSW (SMA), vacuum oscillations (VO) and low-Δm^2 (LOW) solutions [5]. The LOW solution has a low probability and is often excluded from discussions. The remaining mixing angle θ_{13} which determines the element U_{e3} of the lepton mixing matrix is the least known one: there are only upper limits on its value, the most stringent one coming from the CHOOZ reactor neutrino experiment [6]. Together with the solar neutrino observations it gives, for $\Delta m^2_{32} = (2-6) \cdot 10^{-3}$ eV^2,

$$|\sin\theta_{13}| \equiv |U_{e3}| \leq (0.22 - 0.14) \tag{1}$$

The probabilities of the long baseline $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations and subdominant $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations of atmospheric neutrinos depend sensitively on $\sin\theta_{13}$, and therefore knowledge of its value at least by an order of magnitude would be very helpful for planning future long baseline experiments. Yet, the upper limit (1) does not tell us what this value is – it can equally well be just below the upper bound or many orders of magnitude smaller.

In the present letter we show how one can extract information on the value of U_{e3} from the solar neutrino data under the assumption that there is no fine tuning between certain entries of the neutrino mass matrix. We shall derive predictions for U_{e3} corresponding to each one of the neutrino oscillation solutions to the solar neutrino problem.

2. In the three-flavour framework, assuming the hierarchy $\Delta m^2_{21} \ll \Delta m^2_{32}$, the survival probability of the solar ν_e can be written as [7]

$$P_S \simeq c_{13}^4 P + s_{13}^4 \,, \tag{2}$$

where we use the notation $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$ and P is the corresponding survival probability in the two-flavour case which depends on the mixing angle θ_{12} and mass squared difference Δm^2_{21}, with the usual matter-induced potential for neutrinos $V = \sqrt{2} G_F N_e$ [8] replaced by the effective one $V_{eff} = c_{13}^2 V$. It follows from (2) that P_S is rather insensitive to the value of θ_{13} provided that the constraint (1) is satisfied. Therefore the probability of the solar neutrino oscillations cannot be used directly to extract a useful information on U_{e3}. We shall show, however, that such an information can still be obtained from the analyses of the neutrino mass matrix provided that the values of the parameters that govern the solar neutrino oscillations are known.

Assuming the neutrino mass hierarchy $m_1, m_2 \ll m_3$ and $\theta_{23} \simeq 45^\circ$ (which is the best fit value of the Super-Kamiokande data [2]) and taking into account the relative smallness of θ_{13}, it can be shown that in the basis

where the mass matrix of charged leptons is diagonal the neutrino mass matrix m_L must have the approximate form

$$m_L = m_0 \begin{pmatrix} \kappa & \varepsilon & \varepsilon' \\ \varepsilon & 1+\delta-\delta' & 1-\delta \\ \varepsilon' & 1-\delta & 1+\delta+\delta' \end{pmatrix}, \qquad (3)$$

where κ, ε, ε', δ and δ' are small dimensionless parameters. Diagonalization of this matrix [9] yields, in particular,

$$\tan 2\theta_{12} \simeq \frac{(\varepsilon-\varepsilon')}{\sqrt{2}\left[\left(\delta-\frac{\delta'^2}{4}\right)-\left(\frac{\kappa}{2}-\frac{\varepsilon^2+\varepsilon'^2}{4}\right)\right]}, \qquad (4)$$

$$s_{13} \simeq \frac{\varepsilon+\varepsilon'}{2\sqrt{2}}. \qquad (5)$$

We shall be assuming that there are no accidental cancellations between ε and $\pm\varepsilon'$, i.e. that $|\varepsilon+\varepsilon'|$ and $|\varepsilon-\varepsilon'|$ are of the same order of magnitude: $|\varepsilon \pm \varepsilon'| \sim \tilde{\varepsilon}$ where $\tilde{\varepsilon} = max\{|\varepsilon|, |\varepsilon'|\}$. In other words, we assume that $|\varepsilon|$ and $|\varepsilon'|$ are either of the same order of magnitude or one of them is much larger than the other, but bar the possibility that they are equal or approximately equal to each other.

It has been shown in [9] that the MSW effect [8] can only occur for neutrinos, and in particular the LMA and SMA solutions of the solar neutrino problem are only possible, if the parameters of the mass matrix m_L in Eq. (3) satisfy

$$|\tilde{\delta}| \equiv |\delta - \delta'^2/4| > |\kappa/2 - (\varepsilon^2+\varepsilon'^2)/4|. \qquad (6)$$

We shall first assume $|\tilde{\delta}| \gg \varepsilon^2, \varepsilon'^2, |\kappa|$ (our results will also be approximately valid when $\gg$ is replaced by $\gtrsim$). From (4) and (5) one finds

$$\tan 2\theta_{12} \simeq \tilde{\varepsilon}/\sqrt{2}\,\tilde{\delta}, \quad s_{13} \simeq \tilde{\varepsilon}/2\sqrt{2}. \qquad (7)$$

The eigenvalues of the mass matrix m_L can then approximately be written as

$$m_{1,2} \simeq m_0\,\tilde{\delta}\left(1 \pm \sqrt{1+\tan^2 2\theta_{12}}\right), \quad m_3 \simeq 2m_0, \qquad (8)$$

leading, with the identification $\Delta m^2_\odot = \Delta m^2_{21}$, $\Delta m^2_{atm} \simeq \Delta m^2_{32} \simeq (2m_0)^2$, to

$$\tilde{\delta} \simeq \frac{1}{(1+\tan^2 2\theta_{12})^{1/4}}\left(\frac{\Delta m^2_\odot}{\Delta m^2_{atm}}\right)^{1/2}. \qquad (9)$$

From Eq. (7) one then finds

$$s_{13} \simeq \frac{1}{2}\frac{\tan 2\theta_{12}}{(1+\tan^2 2\theta_{12})^{1/4}}\left(\frac{\Delta m^2_\odot}{\Delta m^2_{atm}}\right)^{1/2}. \qquad (10)$$

This expression gives, up to a factor of the order one, the value of the lepton mixing parameter $U_{e3} = s_{13}$ in terms of the parameters describing the solar neutrino oscillations. Substituting the typical values of the parameters that lead to the various neutrino oscillation solutions of the solar neutrino problem [5] we find

$$s_{13} \simeq (0.05-0.15)\ \ (\mathrm{LMA});\quad \sim 10^{-3}\ \ (\mathrm{SMA});$$
$$\sim 10^{-2}\ \ (\mathrm{LOW});\quad \sim 10^{-4}-10^{-3}\ \ (\mathrm{VO}). \qquad (11)$$

Thus, in the case of the LMA solution the value of s_{13} is expected to be only slightly below the CHOOZ limit. The values of s_{13} in this range can lead to observable effects in the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ channels of the long baseline experiments as well as in the subdominant $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ channels of the atmospheric neutrino experiments. They should certainly be detectable in MINOS [10] except perhaps for the values of s_{13} close to the lower border of the allowed region. However, in this case they should still be detectable in the future long baseline experiments with muon storage rings which are being widely discussed now [11]. They may also be detectable in KamLAND [12] and CERN – Gran Sasso [13] experiments provided that the value of s_{13} is close to the upper border of the allowed region for the LMA solution.

In the case of the LOW solution, the values of s_{13} are close to the border of detectability in the experiments with muon storage rings. Whether or not they will be detectable depends on the experimental details which are not yet known. For the SMA and VO solutions of the solar neutrino problem, the predicted values of s_{13} are far too small to lead to observable effects in any of the forthcoming or currently discussed long baseline experiments.

Eq. (10) is not valid when θ_{12} is very close to 45°, namely when $1-\sin^2 2\theta_{12} \lesssim 10^{-5}$. Such a situation can in principle be realized in the case of the VO and LOW solutions of the solar neutrino problem [5,14]. In this case from (8) and (7) one finds $\Delta m^2_\odot \simeq \tilde{\delta}^2 \tan 2\theta_{12}\Delta m^2_{atm} \simeq (\tilde{\delta}\tilde{\varepsilon}/\sqrt{2})\Delta m^2_{atm}$. Our condition $|\tilde{\delta}| \gtrsim \tilde{\varepsilon}^2$ and Eq. (7) then lead to the following upper limit on s_{13}:

$$(s_{13})_{max} \simeq 2^{-4/3}\left(\frac{\Delta m^2_\odot}{\Delta m^2_{atm}}\right)^{1/3}. \qquad (12)$$

Consider now the case $|\tilde{\delta}| \ll \varepsilon^2, \varepsilon'^2, |\kappa|$, i.e. $|\tilde{\delta}| \ll \tilde{\varepsilon}^2$. Condition (6) is then not satisfied and therefore only the VO and LOW solutions to the solar neutrino problem are possible. It is easy to show that in this case s_{13} is also approximately given by Eq. (12) which, however, is now the prediction rather than an upper bound. For typical values of $\Delta m^2_\odot$ relevant for the VO and LOW solutions one then finds $s_{13} \sim 10^{-3}$ and $s_{13} \sim 10^{-2}$ respectively, which are in the same ranges as the values given for these solutions in (11).

3. The above discussion applied to the case of the normal neutrino mass hierarchy, $|m_{1,2}| \ll |m_3|$. Consider now the case of the inverted mass hierarchy with $|m_3| \ll |m_1| \simeq |m_2|$. There are essentially two possibilities. First, the neutrino mass matrix can have the elements

$(m_L)_{12} = (m_L)_{21} \simeq (m_L)_{13} = (m_L)_{31} = m_0$ with the rest of the matrix elements being $\sim 10^{-8} m_0$. Such a matrix can emerge due to an approximate $L_e - L_\mu - L_\tau$ symmetry [15]. It leads to the VO solution of the solar neutrino problem with bi-maximal mixing and $m_1 \simeq -m_2$ (i.e. opposite CP-parities of the mass eigenstates ν_1 and ν_2). The mixing parameter s_{13} is given by the ratio of a combination of the small entries of the mass matrix and m_0 [9], i.e. in this case

$$s_{13} \sim 10^{-8}\,, \tag{13}$$

far too small to be of any practical interest. Second, the neutrino mass matrix may again be of the form (3) with small parameters ε, ε', δ and δ' but now with $\kappa \simeq \pm 2$, which is necessary for the eigenvalues of m_L to satisfy $m_1 \simeq \pm m_2$. As before, one can express s_{13} through the parameters describing the solar neutrino oscillations. Consider first the case $\kappa \simeq 2$, which leads to same sign m_1 and m_2 (same CP-parities of ν_1 and ν_2). In this case any of the neutrino oscillation solutions to the solar neutrino problem can be accommodated. Using the results of [9] one obtains, again up to a factor of the order one,

$$s_{13} \simeq \frac{1}{4}\,\sin 2\theta_{12} \left(\frac{\Delta m^2_\odot}{\Delta m^2_{atm}}\right)\,. \tag{14}$$

The predicted numerical values of s_{13} for various solutions of the solar neutrino problem are

$$s_{13} \simeq (0.15 - 1.5) \times 10^{-2} \ \ (\mathrm{LMA})\,; \ \ \sim 3 \times 10^{-5} \ \ (\mathrm{SMA})\,; \\ \sim 10^{-5} \ \ (\mathrm{LOW})\,; \quad \sim 10^{-8} \ \ (\mathrm{VO})\,, \tag{15}$$

too small to be of interest except perhaps for the LMA case which might lead to observable effects in future experiments with muon storage rings.

Consider now the case $\kappa \simeq -2$, which leads to $m_1 \simeq -m_2$ (opposite CP-parities of ν_1 and ν_2). In this case only the SMA solution to the solar neutrino problem can be accommodated [16]. Diagonalization of the mass matrix yields

$$s_{13} \simeq \tan 2\theta_{12}\,. \tag{16}$$

Since the SMA solution requires $\sin^2 2\theta_{12} \simeq (0.1 - 1) \times 10^{-2}$, this gives

$$s_{13} \simeq 0.03 - 0.1\,, \tag{17}$$

i.e. one can have observable $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations in the long baseline experiments in this case.

4. The results we have obtained rely crucially on the assumption of no fine tuning between certain elements of the neutrino mass matrix. Although we believe that this assumption is natural, such fine tuning is still a possibility; therefore our results should only be considered as the likely values of the parameter U_{e3}.

Eqs. (10), (12) – (14) and (16) are our main results. They give, for various neutrino mass hierarchies and relative CP-parities, the approximate values of the lepton mixing parameter U_{e3} in terms of the values of the parameters governing the oscillations of solar neutrinos. We have checked these relations by direct numerical diagonalization of the neutrino mass matrix m_L for a number of the parameter sets leading to the SMA, LMA, LOW and VO solutions of the solar neutrino problem and found that in most of the cases the agreement was better than 50%.

Our predictions for U_{e3} depend on the assumed hierarchy of neutrino masses. The normal mass hierarchy $|m_{1,2}| \ll |m_3|$ is the most natural one; the mass matrices leading to the inverted mass hierarchy are unstable with respect to small variations of the parameters except in the case when the elements $(m_L)_{12} = (m_L)_{21}$ and $(m_L)_{13} = (m_L)_{31}$ are much larger than the rest of the matrix elements [9]. However, this case leads to the VO solution of the solar neutrino problem with an extremely small U_{e3} of Eq. (13). It should be noted that the question of the neutrino mass hierarchy can in principle be settled experimentally: the long baseline experiments may discriminate between the direct and inverted hierarchies through the earth matter effects on neutrino oscillations.

If the LMA MSW effect proves to be the true solution of the solar neutrino problem, one can expect observable effects in the $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ channels of the long baseline experiments and possibly also in the subdominant $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ channels of the atmospheric neutrino experiments in the most plausible case of the normal neutrino mass hierarchy [17]. In the case of the inverted mass hierarchy, the same is true for the SMA solution. If the VO is established as the true solution, we predict no observable $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations in long baseline experiments for either of the mass hierarchies.

Conversely, a measurement of U_{e3} in atmospheric or long baseline accelerator or reactor neutrino experiments would help discriminate between possible oscillation solutions of the solar neutrino problem. At present, the situation with solar neutrinos is rather unclear: different pieces of data (total rates, recoil electron spectrum and day-night effect in Super-Kamiokande) favour different oscillation solutions, and global fits of all the data are of comparable quality for all the solutions except LOW [5]. The data from the long baseline experiments could help clear the situation up. In particular, a positive signal of $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations would disfavour the VO solution of the solar neutrino problem.

Combined data of the solar neutrino and future long baseline experiments may provide information on the neutrino mass hierarchy even in the absence of any data on matter effects in the long baseline experiments. A positive signal of $\nu_e \leftrightarrow \nu_{\mu(\tau)}$ oscillations along with the established LMA solution of the solar neutrino problem would favour the normal hierarchy; if, however, the future solar data prefer the SMA solution, that positive

signal would point towards the inverted neutrino mass hierarchy.

Finally, if the neutrino mass hierarchy is established through the matter effects in the long baseline experiments, combined data of solar and long baseline experiments could allow one to check our assumption of no fine tuning between the elements of the neutrino mass matrix.

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[1] Y. Suzuki, Talk given at the XIX Intern. Symp. on Lepton and Photon Interactions at High Energies, Stanford University, USA, August 9 – 14, 1999.

[2] A. Mann, Talk given at the XIX Intern. Symp. on Lepton and Photon Interactions at High Energies, Stanford University, USA, August 9 – 14, 1999.

[3] LSND Collaboration, C. Athanassopoulos *et al.*, Phys. Rev. Lett. **81**, 1774 (1998); Phys. Rev. C **58**, 2489 (1998).

[4] Particle Data Group, C. Caso et al., Eur. Phys. J. C **3**, 1 (1998).

[5] For recent analyses of the solar neutrino data and allowed ranges of the parameters see J. N. Bahcall, P. I. Krastev, A. Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998); M. C. Gonzalez-Garcia, P. C. de Holanda, C. Pena-Garay, J. W. F. Valle, hep-ph/9906469; J. N. Bahcall, P. I. Krastev, A. Yu. Smirnov, Phys. Rev. D **60**, 093001 (1999).

[6] CHOOZ Collaboration, M. Apollonio *et al.*, hep-ex/9907037.

[7] C.-S. Lim, preprint BNL-39675, 1987.

[8] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985).

[9] E. Kh. Akhmedov, hep-ph/9909217, to be published in Phys. Lett. B.

[10] MINOS Collaboration, preprint NuMI-L-375, 1998.

[11] See, e.g., A. De Rújula, M. B. Gavela and P. Hernández, Nucl. Phys. B **547**, 21 (1999); M. Campanelli, A. Bueno and A. Rubbia, hep-ph/9905240; V. Barger, S. Geer and K. Whisnant, hep-ph/9906487; O. Yasuda, hep-ph/9910428; I. Mocioiu and R. Shrock, hep-ph/9910554; V. Barger, S. Geer, R. Raja and K. Whisnant, hep-ph/9911524.

[12] P. Alivisatos et al., "KamLAND Proposal", preprint Stanford-HEP-98-03.

[13] ICARUS Collaboration, F. Ameodo *et al.*; NOE Collaboration, M. Ambrosio *et al,* ICANOE proposal, INFN/AE-99-17, CERN/SPSC 99-25, SPSC/P314.

[14] For recent discussions of the LOW solution, see A. de Gouvêa, A. Friedland and H. Murayama, hep-ph/9910286; G.L. Fogli, E. Lisi, D. Montanino and A. Palazzo, hep-ph/9910387.

[15] R. Barbieri, L. J. Hall, D. Smith, A. Strumia, and N. Weiner, JHEP **9812**, 017 (1998).

[16] One could also have the other solutions of the solar neutrino problem in this case provided that the parameters ε and ε' are not small and satisfy $\varepsilon' \simeq \varepsilon$ [9]. However, this is exactly the kind of the relationships between the matrix elements of m_L that we exclude from our consideration.

[17] In the case of the LMA solution, there is also another link between the solar and atmospheric neutrino experiments – the probabilities of the atmospheric neutrino oscillations can depend on $\Delta m^2_\odot$ since the corresponding contribution to the oscillation phase is non-negligible. See O. L. G. Peres and A. Yu. Smirnov, Phys. Lett. B **456**, 204 (1999).